Novel high-gradient accelerating structures

- **Monday**
  Introduction to accelerating structures
  Traditional [iris-loaded] accelerating structures
  Accelerating structure parameters
  Introduction to simulations
  <u>Computer Lab:</u> Basic examples of simulations (MW simulations)

- **Tuesday**
  Dielectric loaded accelerating (DLA) structures
  Wakefield Acceleration
  Discussion of DLA simulations (MW)
  <u>Computer Lab:</u> Basic examples of simulations

- **Wednesday**
  Power Extraction. Two beam acceleration
  Particle studio examples
  <u>Computer Lab:</u> Simulation of DLA. Optimization
  <u>Computer Lab:</u> Introduction to Photonic Crystals (simulations)

- **Thursday**
  Photonic Crystals General Theory
  Photonic Band Gap (PBG) accelerating structures
  <u>Computer Lab:</u> MW Simulations of PBG
  <u>Computer Lab:</u> Final project (simulation / optimization of accelerating structure)

- **Friday**
  Special topics: Beam Break UP. High order mode suppression
  Exotic structures. Metamaterials for accelerator applications
  Conclusions and final project results

Specifics for this course

- No beam dynamics (until Friday). *Almost* all the time we treat the beam as one macro particle
- High gradient structures
- Emphasis on simulations
- We can adjust the material based on *your* feedback
-

Introduction to accelerating structures

Sergey Antipov and Chunguang Jing

outline

- Brief Introduction
- Electrodynamics
  - Maxwell’s equations. Waveguides. Eigenmodes.
  - Dispersion curves. S-parameters. Fourier transforms.
  - Particle interaction. Cherenkov radiation.
- Standard accelerating structures
  - The Floquet theorem. Iris – loaded waveguide
  - Accelerating parameters

Accelerators

Table 1.1. Worldwide inventory of accelerators, in total 15,000.

Category	Number
Ion implanters and surface modifications	7000
Accelerators in industry	1500
Accelerators in non-nuclear research	1000
Radiotherapy	5000
Medical isotopes production	200
Hadron therapy	20
Synchrotron radiation sources	70
Nuclear and particle physics research	110

*U. Amaldi Europhysics News, June 31, 2000

Examples of Accelerator application

- Ion implantation for semiconductors
- Harden cutting tools / reduction of friction in metal parts, bio-materials and implants
- Deep welding for dissimilar metals
- Precision cutting and drilling
- Medical product sterilization
- Food and waste irradiation
- Cross – linking and polymerization
- Waste water remediation
- Cargo examination
- Oil well logging
- Radioisotope production (calibration, medical)
- Synchrotron radiation
- Gemstone color enhancement (topaz, diamonds)

>99% of accelerators are in industry

*Robert Hamm, 2010 at High Energy Physics Advisory Panel

Maxwell's equations

$$\begin{cases} \nabla \cdot \overrightarrow{B} = 0 \\ \nabla \cdot \overrightarrow{D} = 4\pi\rho \\ \nabla \times \overrightarrow{E} = -\frac{1}{c}\frac{\partial \overrightarrow{B}}{\partial t} \\ \nabla \times \overrightarrow{H} = \frac{4\pi}{c}\overrightarrow{j} + \frac{1}{c}\frac{\partial \overrightarrow{D}}{\partial t} \end{cases}$$

Assume: $\overrightarrow{B} = \mu \overrightarrow{H}$ $\quad \overrightarrow{D} = \epsilon \overrightarrow{E}$

$$\nabla \times \nabla \times \overrightarrow{E} = -\frac{\mu}{c}\frac{\partial \nabla \times \overrightarrow{H}}{\partial t}$$

$$\nabla \times \nabla \times \overrightarrow{E} = \nabla\left(\nabla \cdot \overrightarrow{E}\right) - \triangle \overrightarrow{E} =$$

$$= -\frac{\mu}{c^2}\frac{\partial\left(\frac{4\pi}{c}\overrightarrow{j} + \frac{\epsilon}{c}\frac{\partial \overrightarrow{E}}{\partial t}\right)}{\partial t}$$

$$\Delta \overrightarrow{E} - \frac{\epsilon\mu}{c^2}\frac{\partial^2 \overrightarrow{E}}{\partial t^2} - \nabla\left(\nabla \cdot \overrightarrow{E}\right) - \frac{4\pi\mu}{c^2}\frac{\partial \overrightarrow{j}}{\partial t} = 0$$

Make component-differentiation (rot rot → grad div - laplacian)

Boundary conditions. Material equations.

- Boundary conditions

$$B_{2n} - B_{1n} = 0$$

$$D_{2n} - D_{1n} = 4\pi\sigma$$

$$E_{2\tau} - E_{1\tau} = 0$$

$$\left(\overrightarrow{H}_2 - \overrightarrow{H}_2\right) \times \overrightarrow{n} = \frac{4\pi \overrightarrow{i}}{c}$$

Vacuum – Metal boundary - ?

- Material equations
  - Isotropic, linear: $\overrightarrow{D} = \epsilon \overrightarrow{E}$
  - Anisotropic: $\overrightarrow{D} = \widehat{\epsilon} \overrightarrow{E}$
  - Dispersion: $\overrightarrow{D} = \left(1 - \frac{\omega_p^2}{\omega^2}\right) \overrightarrow{E}$
  - Nonlinear: $\overrightarrow{D} = \epsilon \overrightarrow{E} + \epsilon_{NL} \left| \overrightarrow{E}^2 \right| \overrightarrow{E}$

No sources. Plane waves

$$\triangle \overrightarrow{E} - \frac{\epsilon\mu}{c^2}\frac{\partial^2 \overrightarrow{E}}{\partial t^2} = 0$$

$$\overrightarrow{A} e^{-i\omega t + ik_z z}$$

Observe, that this would satisfy equation

$$-k_z^2 \overrightarrow{A} + \frac{\epsilon\mu}{c^2}\overrightarrow{A} = 0$$

$$k_z^2 = \frac{\epsilon\mu\omega^2}{c^2} = \epsilon\mu k_0^2$$

Provided, there is a restriction on k_z and ω

Effective wavelength:

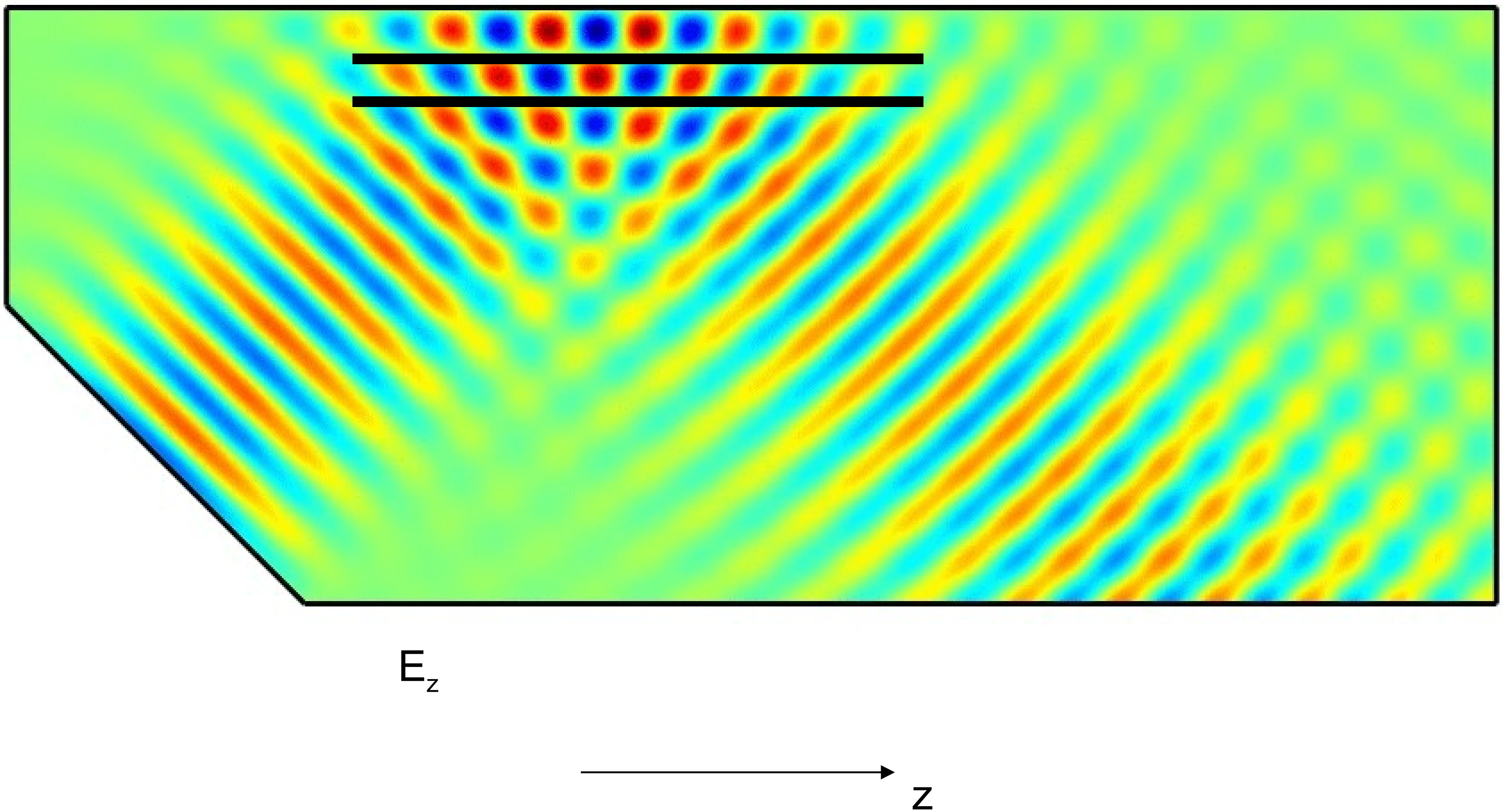
$$cos\,(k_z \lambda_{eff}) = 0$$

$$\lambda_{eff} = \frac{2\pi}{k_z}$$

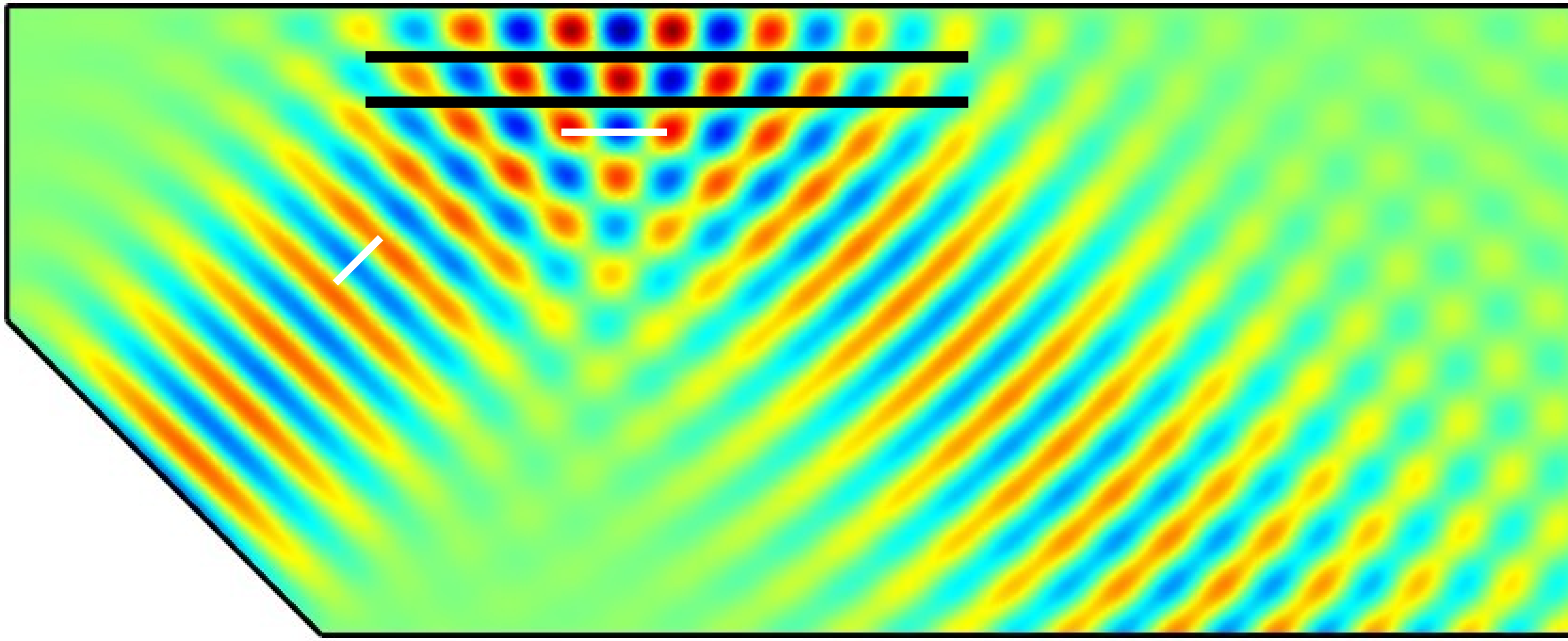
Free – space wavelength

$$\lambda_0 = \frac{2\pi}{k_0} = \frac{c}{f}$$

Waveguiding

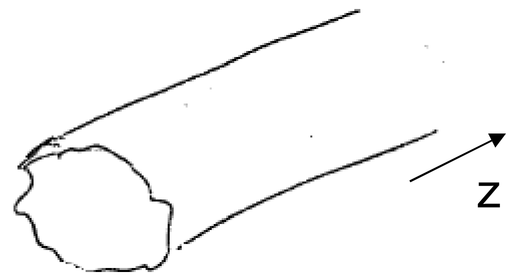


waveguiding



Waveguiding

$$\triangle \overrightarrow{E} - \frac{\epsilon\mu}{c^2}\frac{\partial^2 \overrightarrow{E}}{\partial t^2} = 0$$



Boundary condition

$$E_{z\perp} = 0$$

Again, searching for propagating solution

$$\overrightarrow{E} = \overrightarrow{E_\perp}(x, y)\, e^{-i\omega t + ik_z z}$$

$$\triangle_\perp \overrightarrow{E_\perp} - k_z^2 \overrightarrow{E_\perp} + \epsilon\mu k_0^2 \overrightarrow{E_\perp} = 0$$

For z – component we get

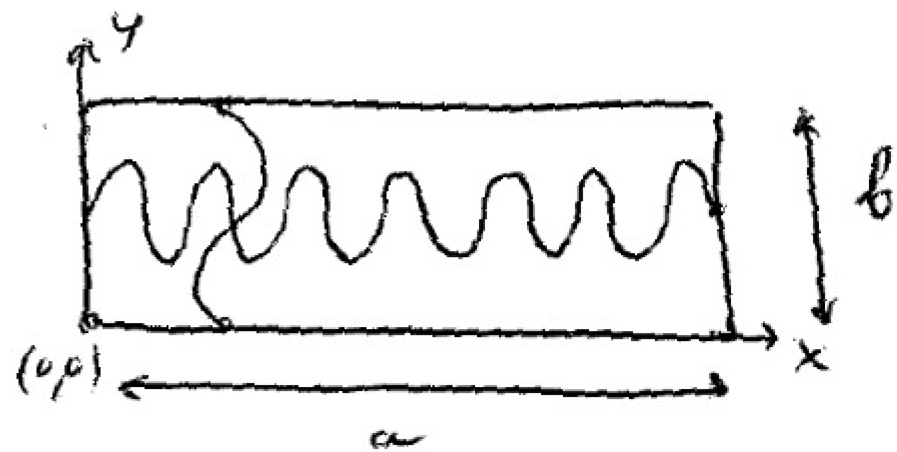
$$\triangle_\perp E_{z\perp} - k_z^2 E_{z\perp} + \epsilon\mu k_0^2 E_{z\perp} = 0$$

$$\boxed{\triangle_\perp E_{z\perp} = \left(k_z^2 - \epsilon\mu k_0^2\right) E_{z\perp}}$$

Has infinitely many solutions, but there is a way to count (classify) them

Example: Rectangular waveguide

$$\Delta_\perp E_z + \chi^2 E_z = 0$$



$$E_z = X(x) \cdot Y(y)$$

$$\frac{\partial^2 X}{\partial t^2} + {\chi_x}^2 X = 0$$

$$E_z = E_0 sin\left(\frac{m\pi}{a}x\right)\left(\frac{n\pi}{b}y\right)$$

$$\chi^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Fourier row analogy

Eigenmodes

$$\Delta_\perp E_{zi} + {\chi_i}^2 E_{zi} = \Delta_\perp E_{zi} - \lambda_i E_{zi} = 0$$

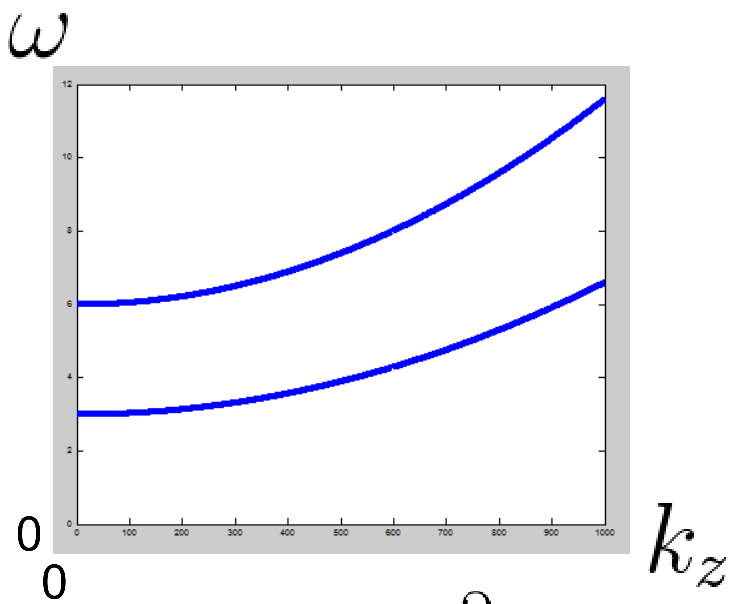
$$\left({k_z}^2 - \epsilon\mu {k_0}^2 + {\chi_i}^2\right) E_{zi} = 0$$

TM $H_z = 0$ vs TE modes

Dispersion $${k_z}^2 = \epsilon\mu {k_0}^2 - {\chi_i}^2$$ For *i*-th mode

What happens if arbitrary field distribution launched from one end into the waveguide at frequency f?

Dispersion plot

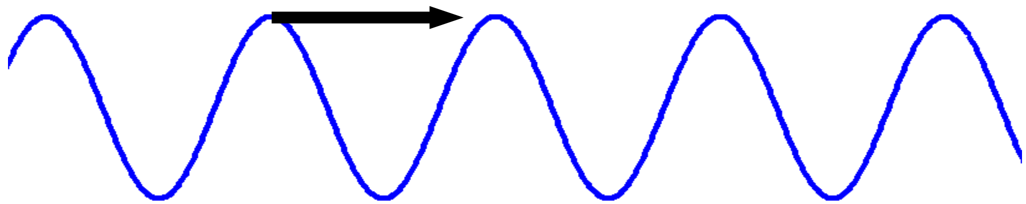


$$k_z{}^2 = \epsilon\mu k_0{}^2 - \chi_i{}^2$$

$$k_0 = \frac{\omega}{c}$$

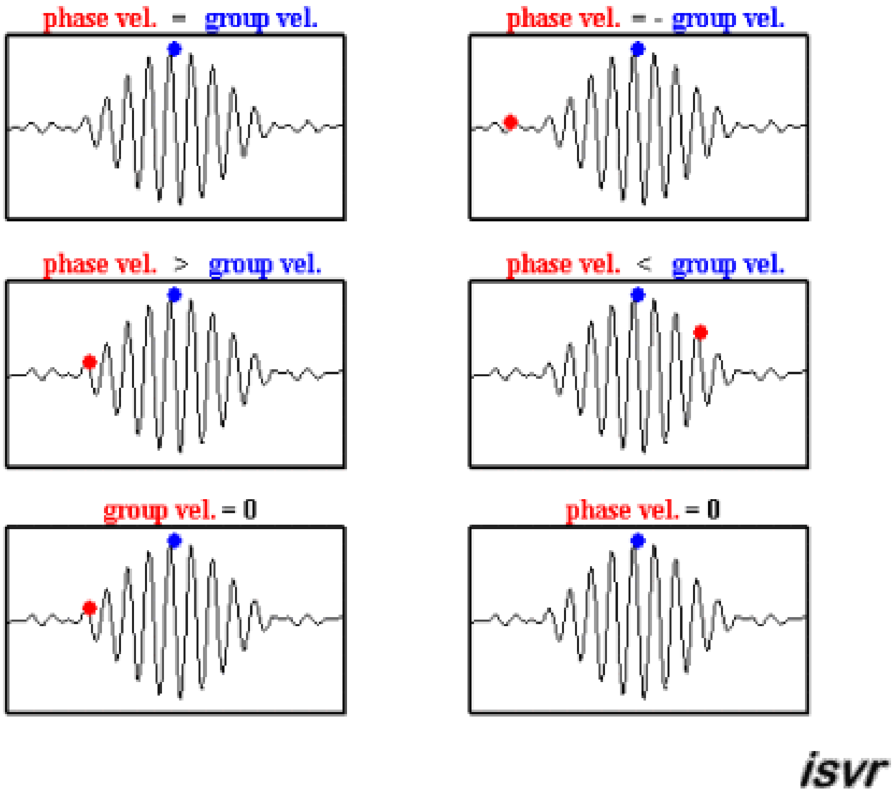
$$v_{group} = \frac{\partial\omega}{\partial k_z}$$

$$\lambda_{eff} = \frac{2\pi}{k_z} \qquad T = 2\pi/\omega$$



$$v_{ph} = \omega/k_z$$

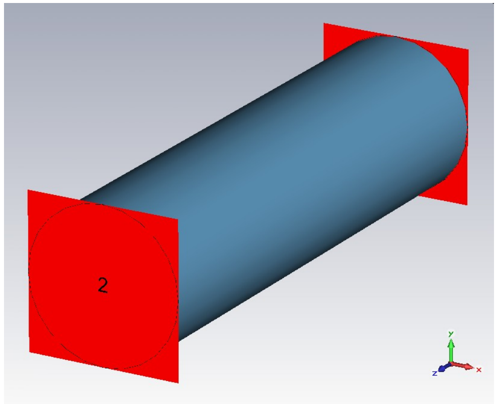
Phase velocity, group velocity

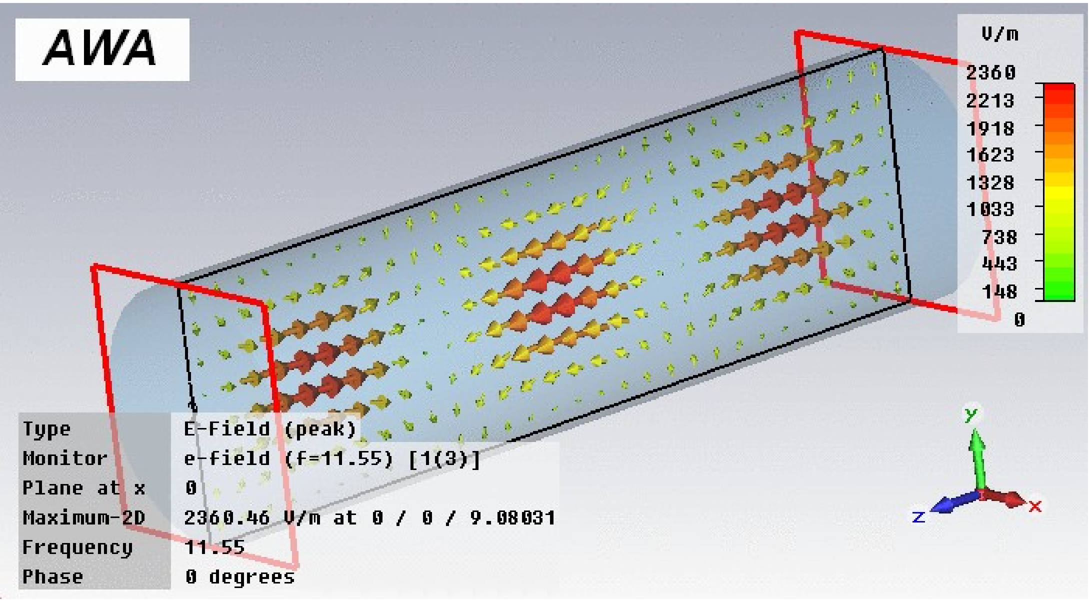


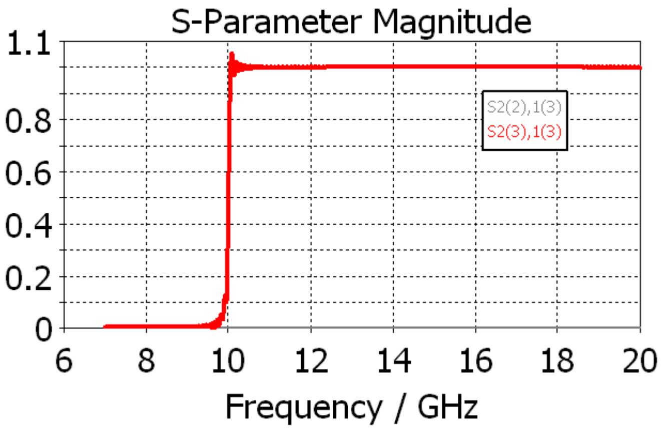
http://www.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-further-dispersive.htm

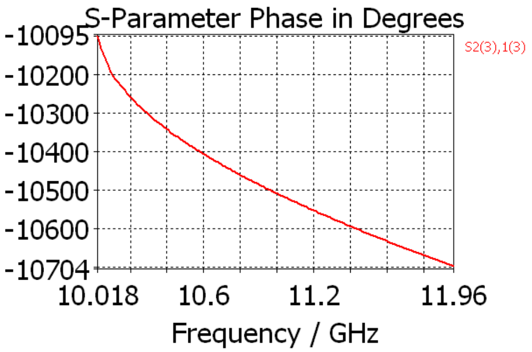
Simulations

- Transmission, S21 / Reflection, S11
- Phase advance $\lambda_{eff} = \frac{2\pi}{k_z}$ vs structure length



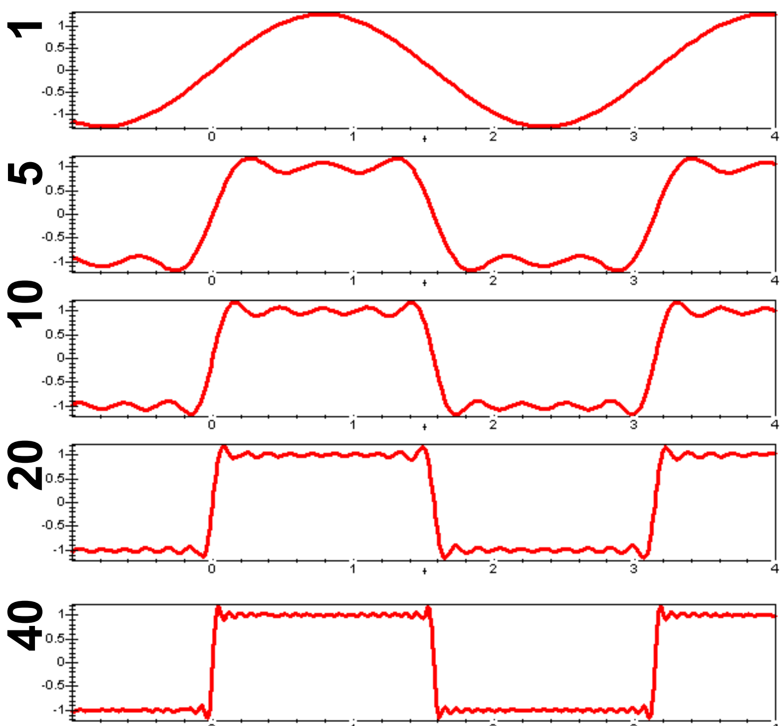




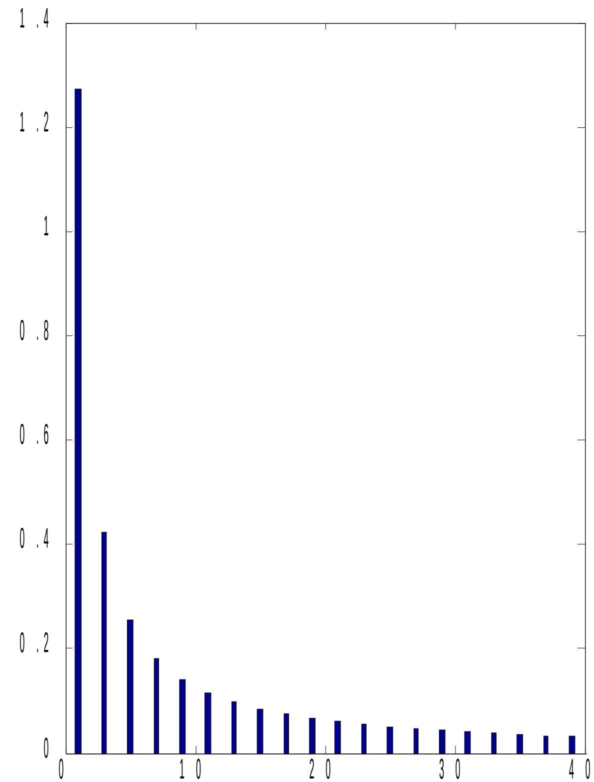


Fourier transform (example: row)

Original periodic signal (bunch train)



spectrum



F(t)=1.273*sin(1*w*t)+0.424*sin(3*w*t)+0.255*sin(5*w*t)+0.182*sin(7*w*t)+0.141*sin(9*w*t)+0.116*sin(11*w*t)+0.098*sin(13*w*t)
+0.085*sin(15*w*t)+0.075*sin(17*w*t)+0.067*sin(19*w*t)+0.060*sin(21*w*t)+0.055*sin(23*w*t)+0.051*sin(25*w*t)
+0.047*sin(27*w*t)+0.044*sin(29*w*t)+0.041*sin(31*w*t)+0.039*sin(33*w*t)+0.036*sin(35*w*t)+0.034*sin(37*w*t)
+0.033*sin(39*w*t)

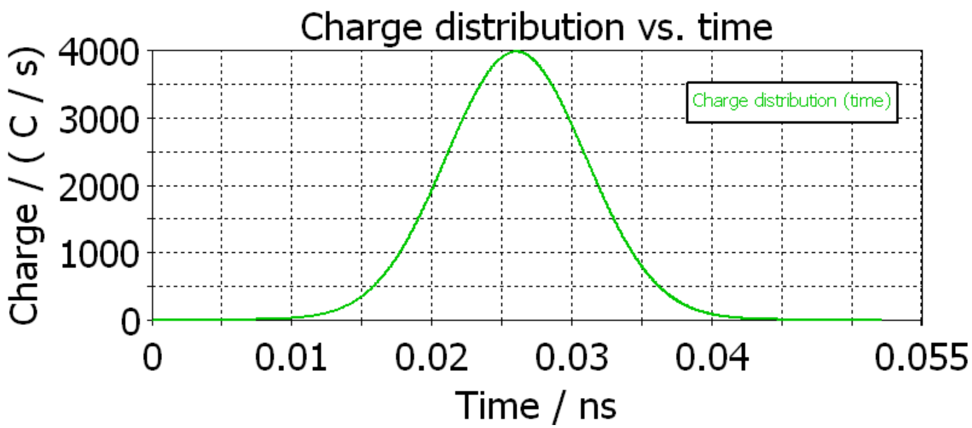
Fourier transform

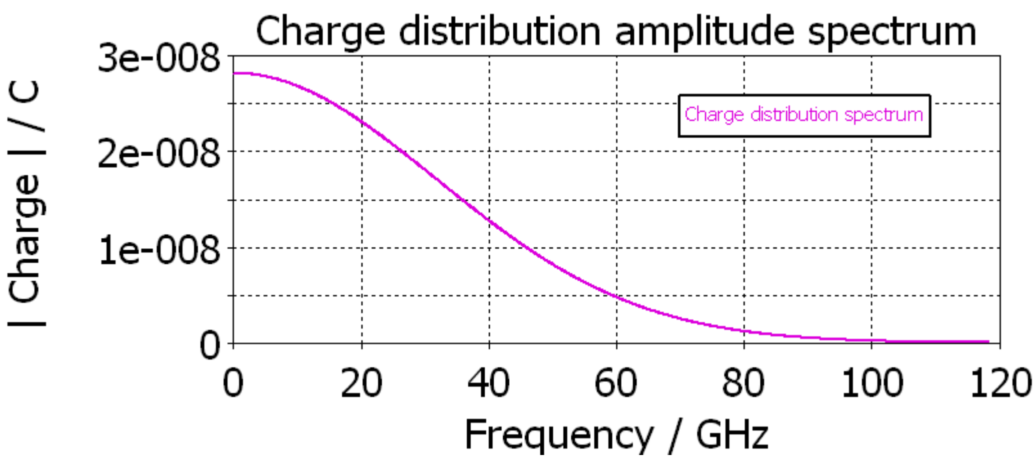
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$$

$$j_z = \frac{q}{2\pi} v \cdot Tr(x,y) \cdot \delta(z - vt)$$

$$j_z = \frac{q}{2\pi} Tr(x,y) \cdot e^{i\frac{\omega}{v}z}$$





Particle enforces not only frequency, but k_z as well

Phase matching

Delta function / gaussian

Frequency content

Applications in wakefield detection

Applications in simulations

Waveguide + dielectric + beam

$$\Delta E_z - \frac{\epsilon\mu}{c^2}\frac{\partial^2 E_z}{\partial t^2} = \frac{4\pi}{\epsilon}\nabla\rho + \frac{4\pi\mu}{c^2}\frac{\partial j_z}{\partial t}$$

Processes of form:

$$e^{-i\omega t + ik_z(\omega)z}$$

Processes of form:

$$e^{-i\omega t + ik_e(\omega)z}$$

For the particle beam

$$\rho, j_z \sim f(z - vt)$$

$$e^{ik_e(z-vt)z}$$

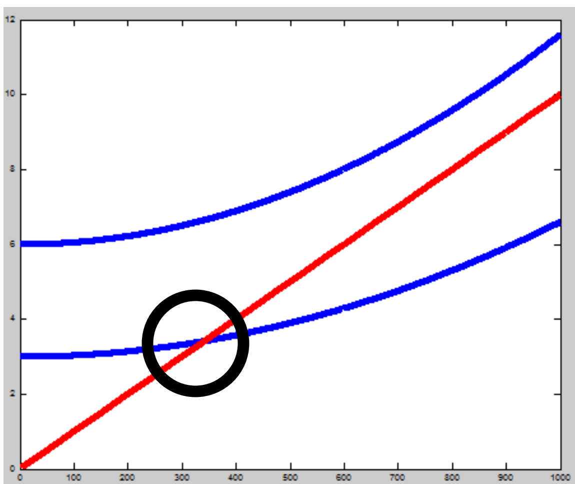
$${k_z}^2 = \epsilon\mu {k_0}^2 - {\chi_i}^2$$

AND

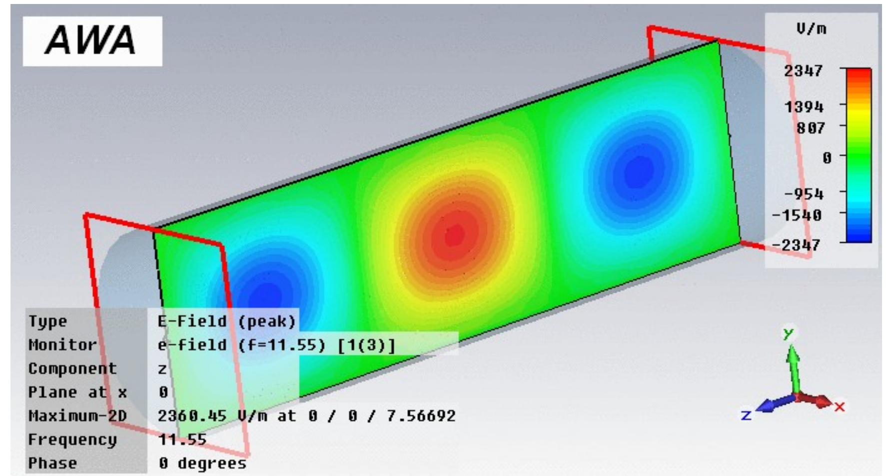
$$k_e = \omega/v$$

Synchronism

- Phase velocity of the mode should match velocity of a particle



Wakefield

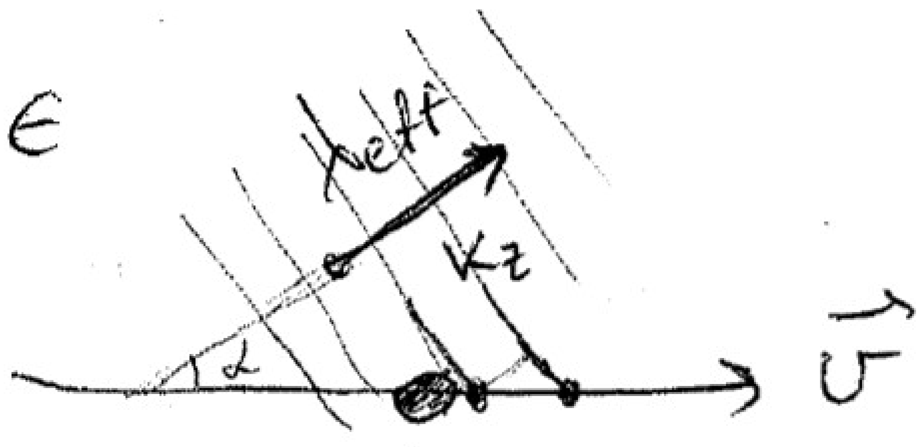


$$k_z{}^2 = \epsilon\mu k_0{}^2 - \chi_i{}^2$$

$$k_z{}^2 = \frac{\omega^2}{v^2} \qquad v = \beta c$$

Vacuum – no interaction $\quad \epsilon = \mu = 1$

Cherenkov radiation



$$v = \beta c$$

$$\lambda_{trajectory} = \frac{\lambda_{eff}}{cos\alpha} = \frac{2\pi}{k_z cos\alpha} = \frac{2\pi c}{\epsilon^{1/2} cos\alpha \cdot \omega} = vT$$

$$cos\alpha = \frac{1}{\epsilon^{1/2}\beta}$$

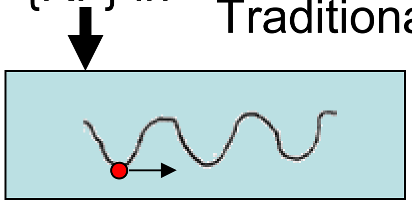
There is a restriction on angle for radiation to occur

Accelerating process

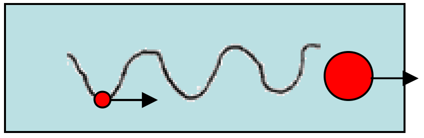
- Particle [electron] energy, velocity (≈ c)
- Accelerating structure, that supports mode
- Mode has to have right field pattern
- Mode has to provide synchronization
- Particle interaction with the mode

{RF} in

Traditional scheme



Wakefield acceleration



Standing wave / traveling wave

{RF} source, Coupling

Drive beam,

timing

Iris-loaded, DLA, PBG, Plasma, Laser-structure ...

What to do with empty waveguide?

empty: $k_{zm}^2 = k_0^2 - \chi_m^2$ $\qquad k_0 = \omega / c$

$$k_z{}^2 = \frac{\omega^2}{v^2} \quad \text{(particle)}$$

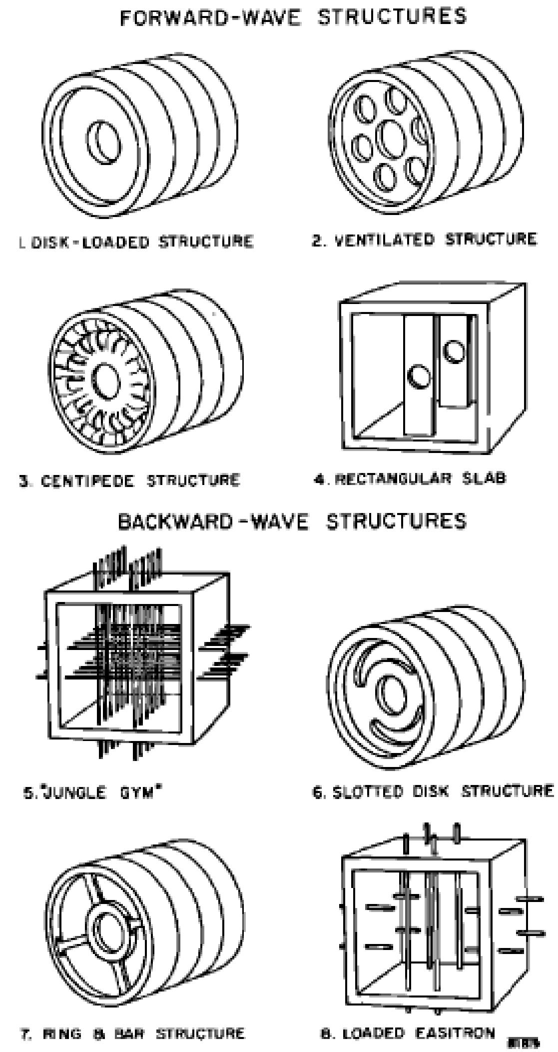
Phase velocity

$$v_{ph} = \frac{\omega}{k_{zm}} = \frac{\omega}{\sqrt{\frac{\omega^2}{c^2} - \chi_m^2}} = \frac{c}{\sqrt{1 - \frac{\chi_m^2 c^2}{\omega^2}}} > c$$

Need to slow mode down

Introduce **any** periodicity

Figure 6-19 Slow-wave structures proposed for linear accelerators.



The Floquet theorem. Dispersion

- Instead of [empty waveguide]

$$\vec{E} = \vec{E}_\perp(x, y)e^{-i\omega t + ik_{z0}z}$$

- We have due to periodicity

$$\vec{E} = \vec{E}_\perp(x, y, z)e^{-i\omega t + ik_{z0}z}$$

-
- Floquet theorem says [*kd* is a phase advance per period]*:*
-

$$E_0(z) = E_0(z + d)e^{\pm ik0d}$$

- Fourier row [space harmonics]
-

$$E(r, z) = \sum_{-\infty}^{\infty} a_n(r)e^{-i\frac{2\pi n}{d}z}$$

- We end up with effective k_z

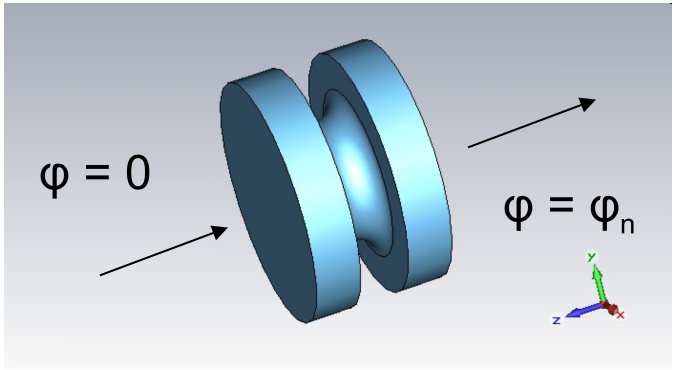
$$\vec{E} = \vec{E}_\perp(x, y)e^{-i\omega t + ik_z z + i\frac{2\pi n}{d}z}$$

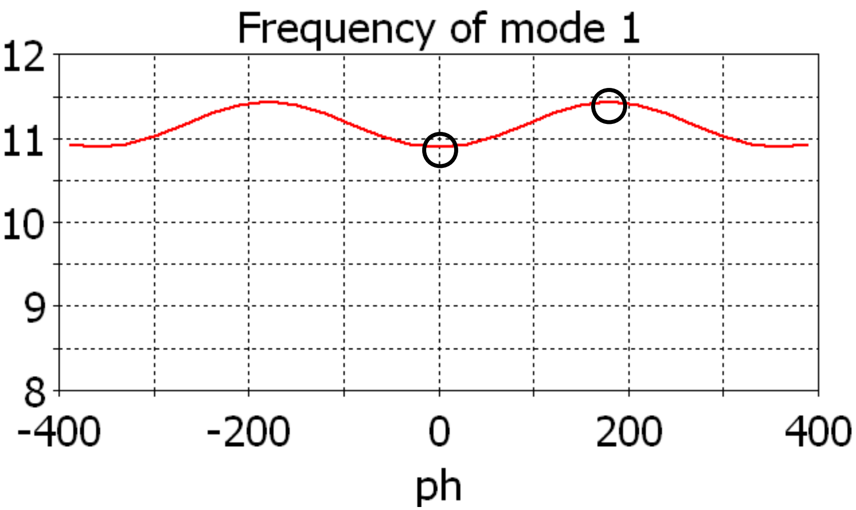
simulations

- Corrugated structure:
- Periodic cells:
  - Phase advance and dispersion
  - Space harmonics

- Eigenmodes.
- Geometry optimization
- All set to calculate accelerating parameters

Dispersion



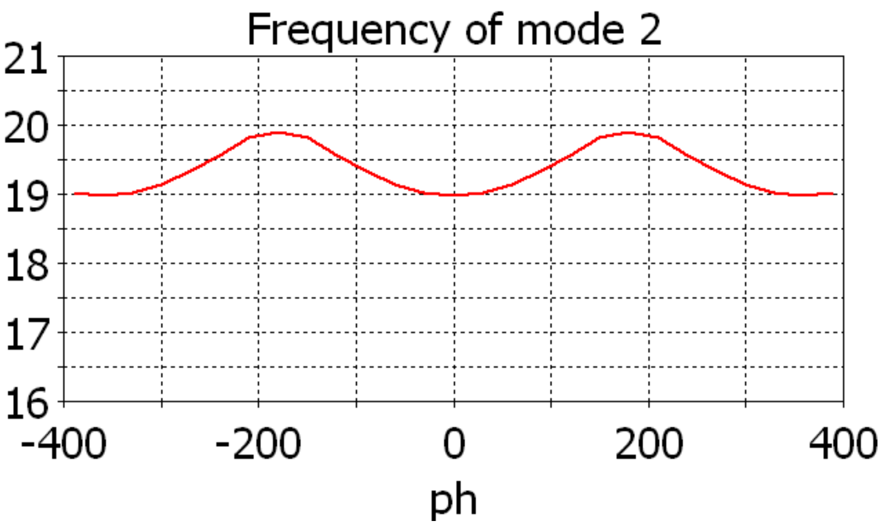


$k_z \cdot d$ – phase advance

Synchronization with particle is guaranteed!

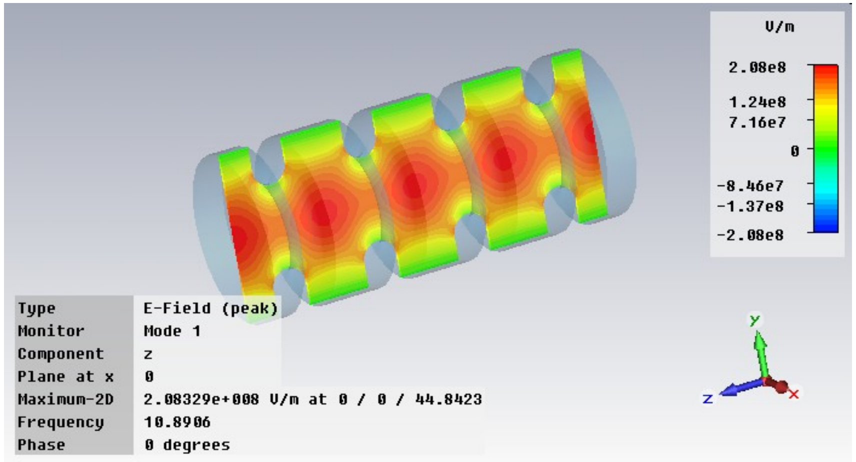
There are forbidden bands. Why?!

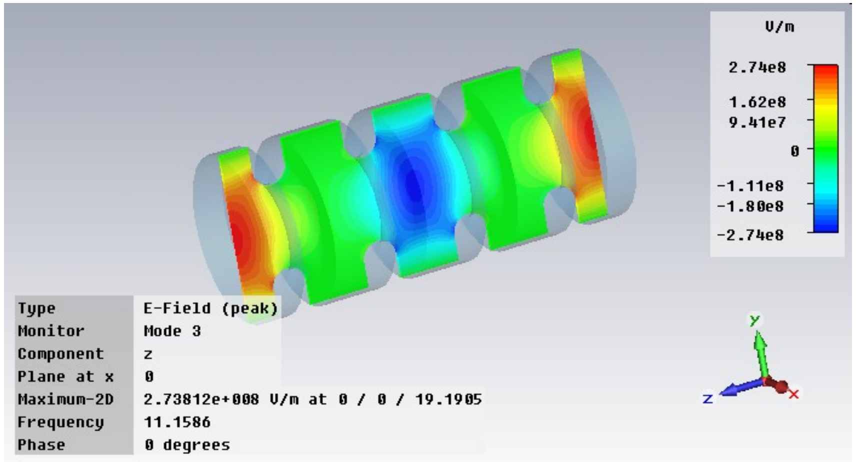
Theme alert ☺

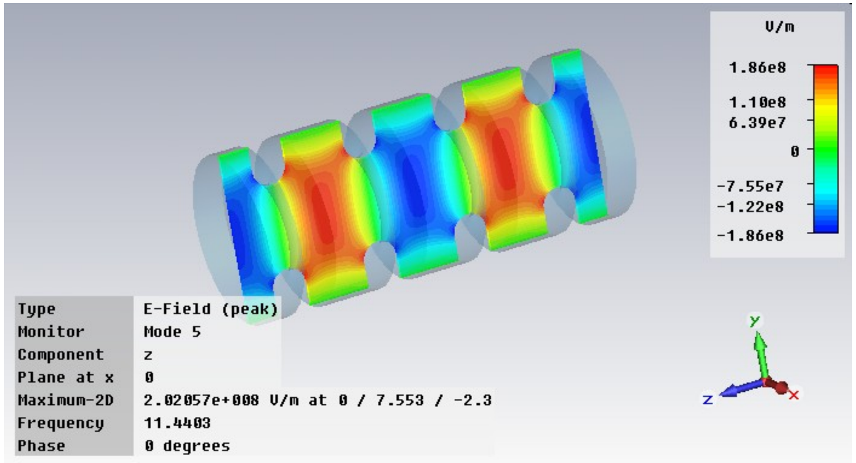


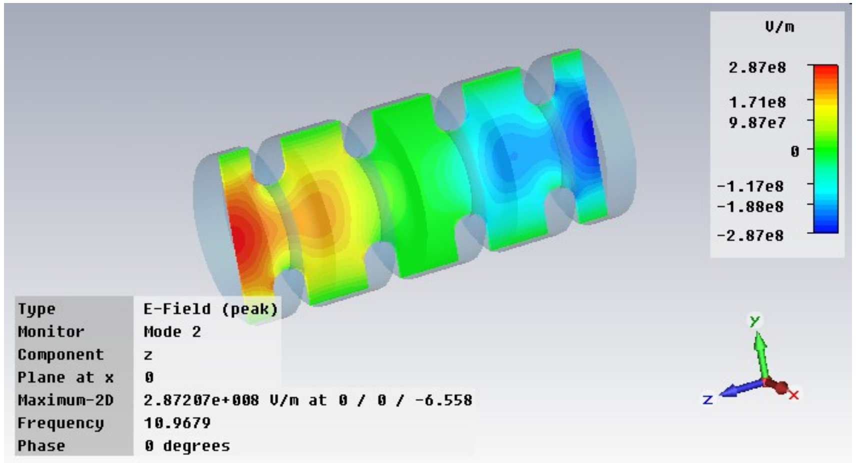
- Travelling wave accelerator vs Standing wave accelerator

Phase advance per cell

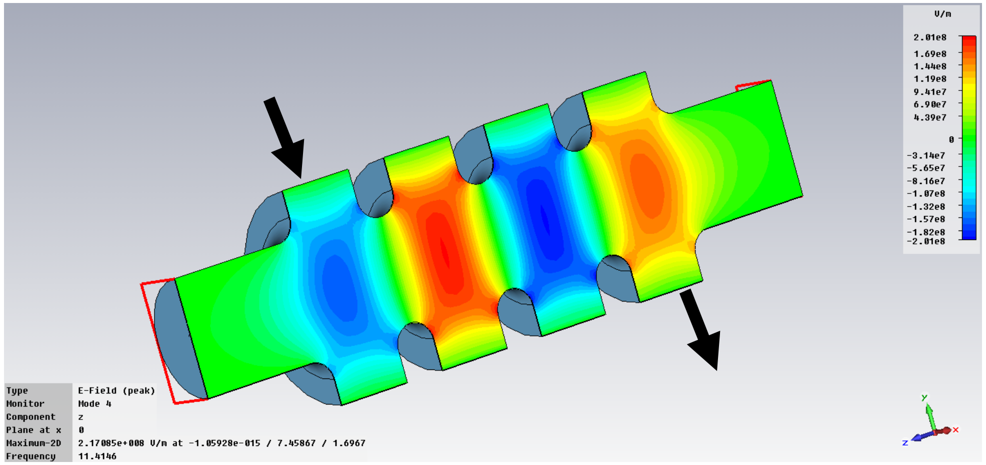






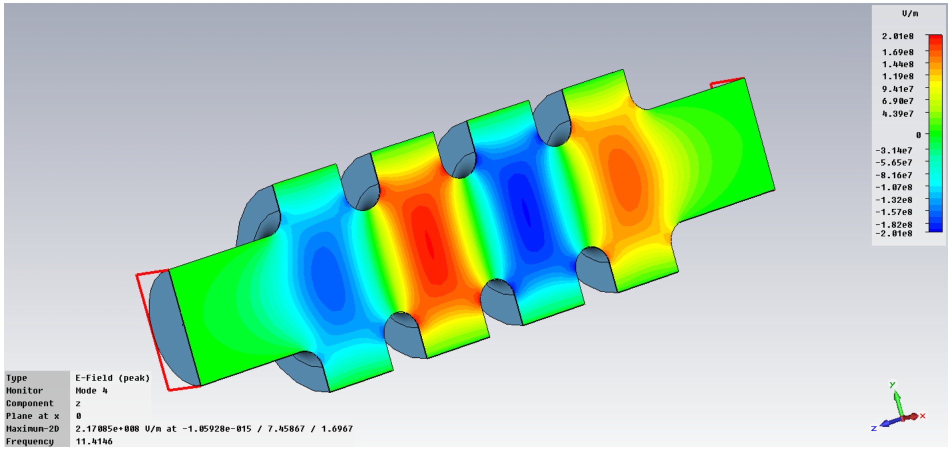


Iris – loaded waveguide



- Corrugation – form of periodicity in this case
- Period – cell length
- Alternative interpretation: coupled pill – box cavities (set of coupled oscillators)
- Travelling wave accelerator vs Standing wave accelerator

Transit effect simulation



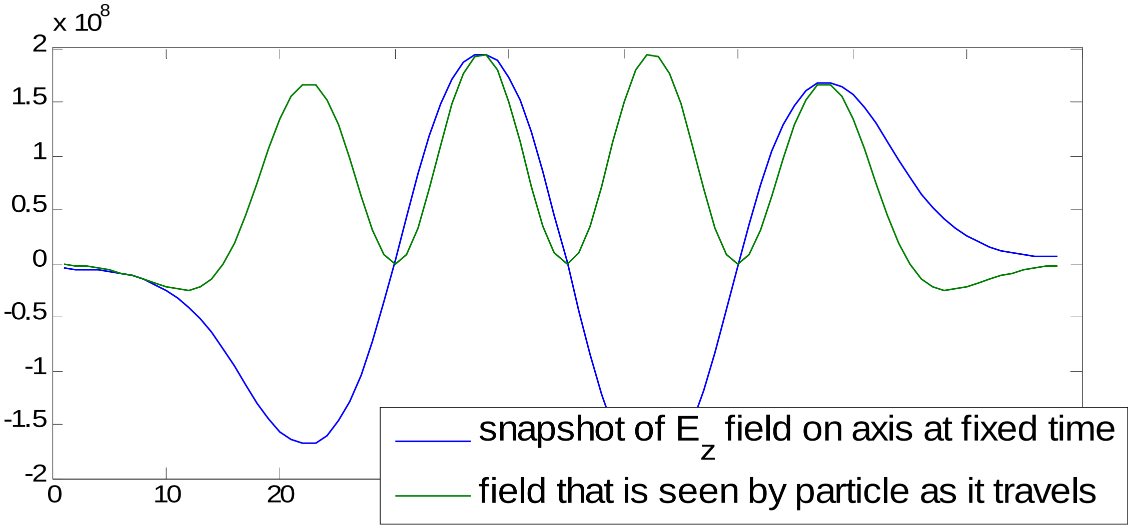
$$\Delta W = q \int_{-L/2}^{L/2} E(0,z) cos(\omega t(z) + \phi) dz$$

$$\omega t = \omega z / v = 2\pi z / \beta\lambda$$

$$T = \frac{\int_{-L/2}^{L/2} E(0,z) cos(2\pi z/\beta\lambda) dz}{\int_{-L/2}^{L/2} E(0,z) dz}$$

Panofsky equation: energy gain

$$\Delta W = q E_0 T cos(\phi L)$$



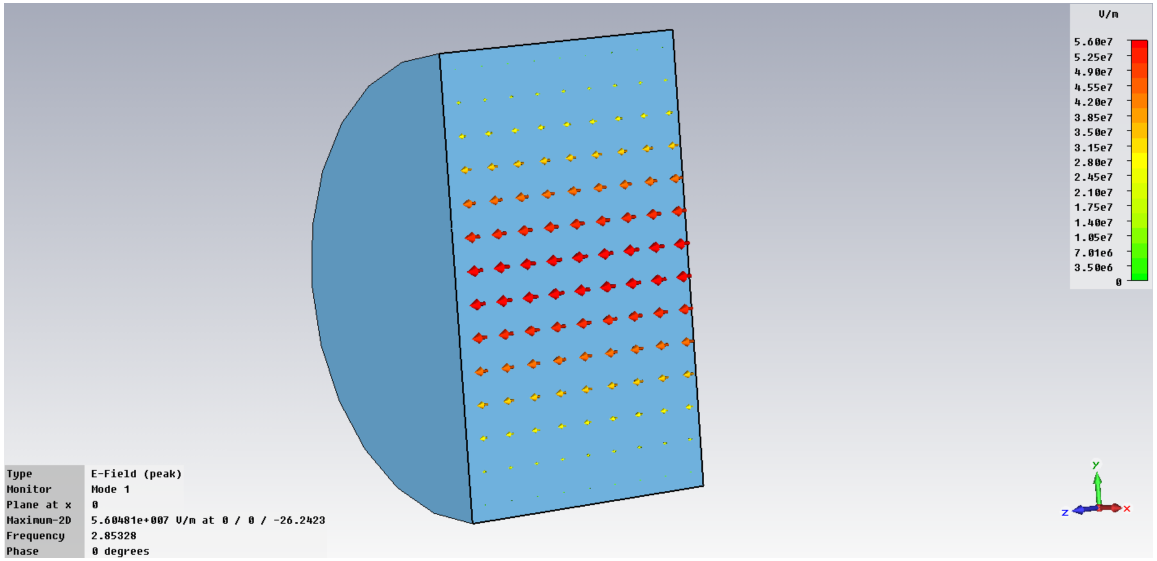
Transient effect simulation

- SW_3_cells_parameters_accelerator.cst
- D=8.7474 mm cell [optimize] for 11.424 GHz for 2π/3
- Template based postprocessing
  - Evaluate field along arbitrary coordinates
  - 1D result from 1D result, rescale
- Export to matlab
- plot(ez(:,1),[-(ez(:,2)) ez(:,2).*scale(:,2)])
- T=sum(ez(:,2).*scale(:,2))/sum(abs(ez(:,2)))

Analogous to:

$$T = \frac{\int_{-L/2}^{L/2} E(0,z) cos(2\pi z/\beta\lambda) dz}{\int_{-L/2}^{L/2} E(0,z) dz}$$

Accelerating parameters of a pillbox SW cavity



TM010 mode @ 2.856 GHz

$\lambda 0$=104.97mm

a=40.1789mm

d=52.4847mm

E0 = 56 MV/m

Cavity parameters

*For TW define per unit length

$$U = \frac{\epsilon}{2}\int_V \vec{E}^2 dV$$ Stored energy = 1 Joule

$$P = \frac{R_s}{2}\int_S H^2 dS$$ Power dissipation = 968 kWatt

$$Q = \frac{\omega U}{P}$$ Quality factor = 18500

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)cos(2\pi z/\beta\lambda)dz}{\int_{-L/2}^{L/2} E(0,z)dz}$$ Transit factor = .64

$T \cdot E_0$		Effective gradient = 35.7 MV/m
$T \cdot E_0 \cdot d$		Panofsky voltage / energy gain = 1.87 MeV
$R_{sh}=(T \cdot E_0 \cdot d)^2/P$	per structure	Effective shunt impedance = 3.6 MOhm
$r_{sh}=(T \cdot E_0 \cdot d)^2/P/d$	per unit length	Effective shunt impedance = 69 MOhm/m
R_{sh}/Q	per structure	Effective R over Q = 196 Ohms
$R_{sh}/Q\ /d$	per unit length	Effective R over Q = 3.7 kOhm/m

Accelerating parameters

- Shunt impedance measures efficiency of acceleration for a given dissipated power
- r / Q shows how much accelerating field one has for a given stored energy; depends on geometry only as loss excluded
-

Computer lab / homework

- Simulate X-Band iris loaded structure
- Optimize geometry to match [$\pi/3$, $2\pi/3$, $\pi/4$ mode] to 11.424 GHz
- Calculate
  - Transit factor
  - Q, R/Q, R
  - …